NAME: ____
(Print) Last Name, First Name

STUDENT NUMBER: ____________________________

The Irving K. Barber School of Arts and Sciences

Physics 231— Winter 2014/2015 – Term 1
FINAL EXAMINATION

Instructor: Jake Bobowski

Tuesday, December 9, 2014 Time: 09:00 - 12:00
Location: ART 386

This Examination was prepared by Jake Bobowski
Not including this coversheet, the exam consists of 15 numbered pages.

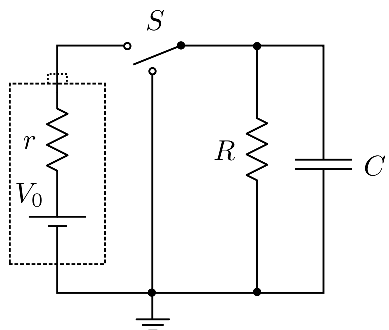
- **Attempt all of problems 1 through 4.**
- **Attempt any three of problems 5 through 8.**

If necessary, you may use the backs of pages for calculations.
You must clearly show your work to receive full credit.
Writing down only the correct final answer will not earn full credit.
Include labelled diagrams whenever appropriate.
Include units with the final answer whenever appropriate.

1	2	3	4	5	6	7	8	total
15	10	10	15	10	10	10	10	80

Free Response: Write out complete answers to the following questions. Show your work.

(15pts) **1.** The circuit below has a battery, a switch S, and a resistor R in parallel with capacitor C. The battery is modelled as a voltage source V_0 in series with resistor r which represents the internal resistance of the battery.



(a) Suppose that the switch S is initially in the down position for a long time and then, at time $t = 0$, it is moved to the up position. If $V_0 = 9.0$ V, $r = 1.5\ \Omega$, $R = 16$ kΩ, and $C = 22\ \mu$F, how long does it take the capacitor acquire a charge of 0.11 mC? (5 marks)

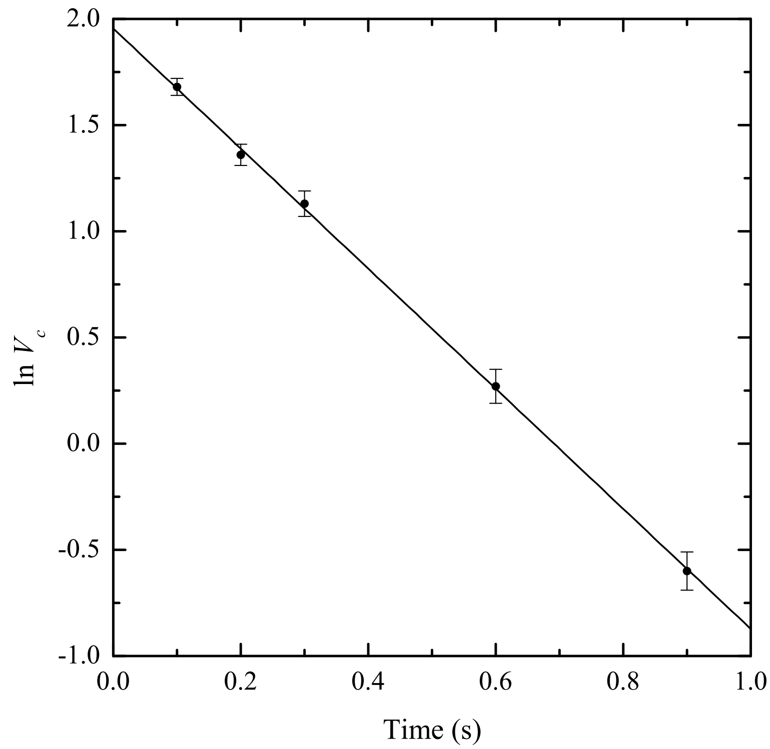
15 pts

(b) Now suppose that the switch has been in the up position for an unknown amount of time. Then, at $t = 0$, it is moved into the down position to discharge the capacitor. The voltage across the capacitor V_c and its uncertainty are measured as a function of time as shown in the table below. Also shown in the table are the values of $\ln V_c$. Complete the table by entering the error in $\ln V_c$ in the final column of the table. Clearly explain how these values are obtained. (5 marks)

t (s)	V_c (V)	ΔV_c (V)	$\ln V_c$	$\Delta(\ln V_c)$
0.1	5.4	0.2	1.68	
0.2	3.9	0.2	1.36	
0.3	3.1	0.2	1.13	
0.6	1.31	0.1	0.27	
0.9	0.55	0.05	−0.60	

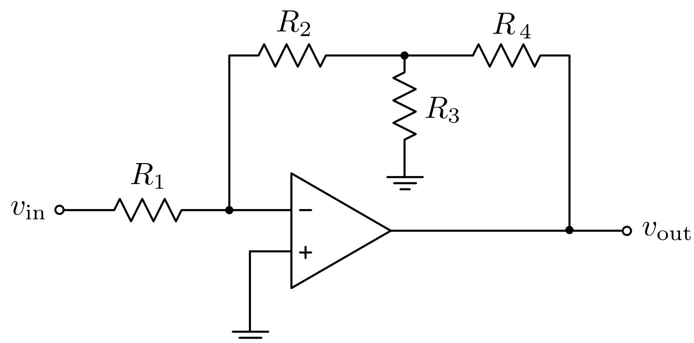
0 pts

(c) The $\ln V_c$ versus t data from the table are plotted below and fit to a straight line. The equation of the best-fit line is $y = mx + b$ with $m = -2.83 \pm 0.05\ \mathrm{s}^{-1}$ and $b = 1.95 \pm 0.02$. Use this information to determine the initial voltage across the capacitor at $t = 0$ and its uncertainty $[V_c(0) \pm \Delta V_c(0)]$ and the time constant of the decay and its uncertainty $[\tau \pm \Delta\tau]$. Does the experimentally determined value of τ agree with the expected value? (5 marks)



0 pts

(10pts) **2.** Consider the following op-amp amplifier circuit:



(a) Find an expression for the gain ($G = v_{\text{out}}/v_{\text{in}}$) of the amplifier in terms of R_1, R_2, R_3, and R_4. (7 marks)

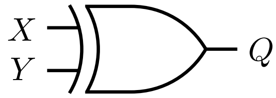
10 pts

(b) What does the gain become if $R_4 = 0$? Does this result agree with what you would expect? Explain. (2 marks)

(c) What is the gain of the amplifier if $R_1 = R_3 = 1$ kΩ and $R_2 = R_4 = 100$ kΩ? (1 mark)

0 pts

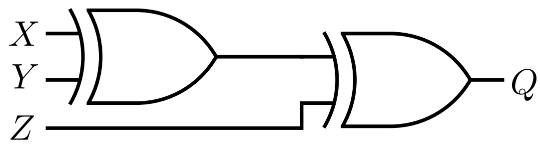
(10pts) **3.** The XOR gate and its truth table are shown below.



X	Y	$Q = X \oplus Y$
0	0	0
0	1	1
1	0	1
1	1	0

The XOR gate can be thought of as a two-input "parity" checker. If an odd number of inputs are high, then the output is high. Alternatively, if zero or an even number of the inputs are high, then the output is low.

(a) Confirm that the circuit below acts as a three-input parity checker. Do this by writing out the complete truth table for this circuit and thus confirm that it works as required. (4 marks)

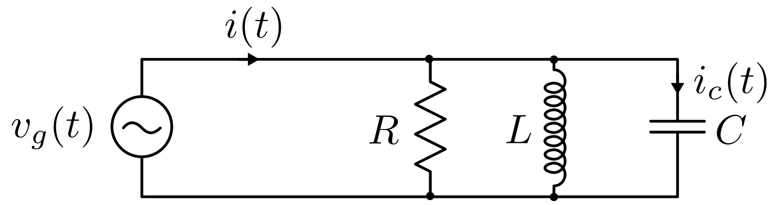


10 pts

(c) Design a four-input parity checking circuit. Construct the complete truth table to confirm that the circuit that you designed works as required. (6 marks)

0 pts

(15pts) **4.** Consider the parallel LRC circuit shown below. The function generator supplies $v_g(t) = V_0 \sin \omega t$ to the circuit.



(a) Find expressions for the amplitude I_0 and phase ϕ of the current $i(t) = I_0 \sin(\omega + \phi)$ shown in the figure above. Give your expressions in terms of R, L, C, ω, and V_0. (8 marks)

15 pts

(b) For values of $\omega \gg 1/\sqrt{LC}$, show that I_0 can be approximated as $I_0 \approx A\omega^n$. Determine A and n. (1.5 marks)

(c) For values of $\omega \ll 1/\sqrt{LC}$, show that I_0 can be approximated as $I_0 \approx B\omega^m$. Determine B and m. (1.5 marks)

(c) Sketch a plot of I_0 versus ω. On your plot, label the point where $\omega = 1/\sqrt{LC}$. (4 marks)

0 pts

Complete any of the **three** remaining problems (5, 6, 7, 8).

Clearly indicate which three problems you wish to be graded by entering three numbers into the table below.

0 pts

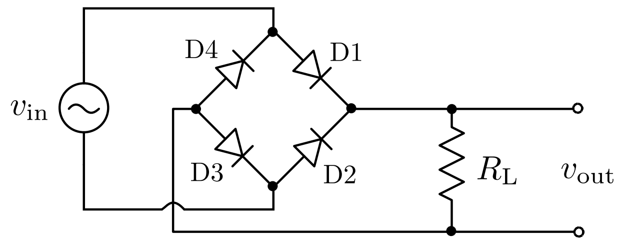
(10pts) **5.** Make use of Euler's equation $\left(e^{\pm j\phi} = \cos\phi \pm j\sin\phi\right)$ to help you evaluate the following integral:

$$\int_0^{\pi/4} e^x \cos x \, dx$$

You must show all of your work to receive full credit.

10 pts

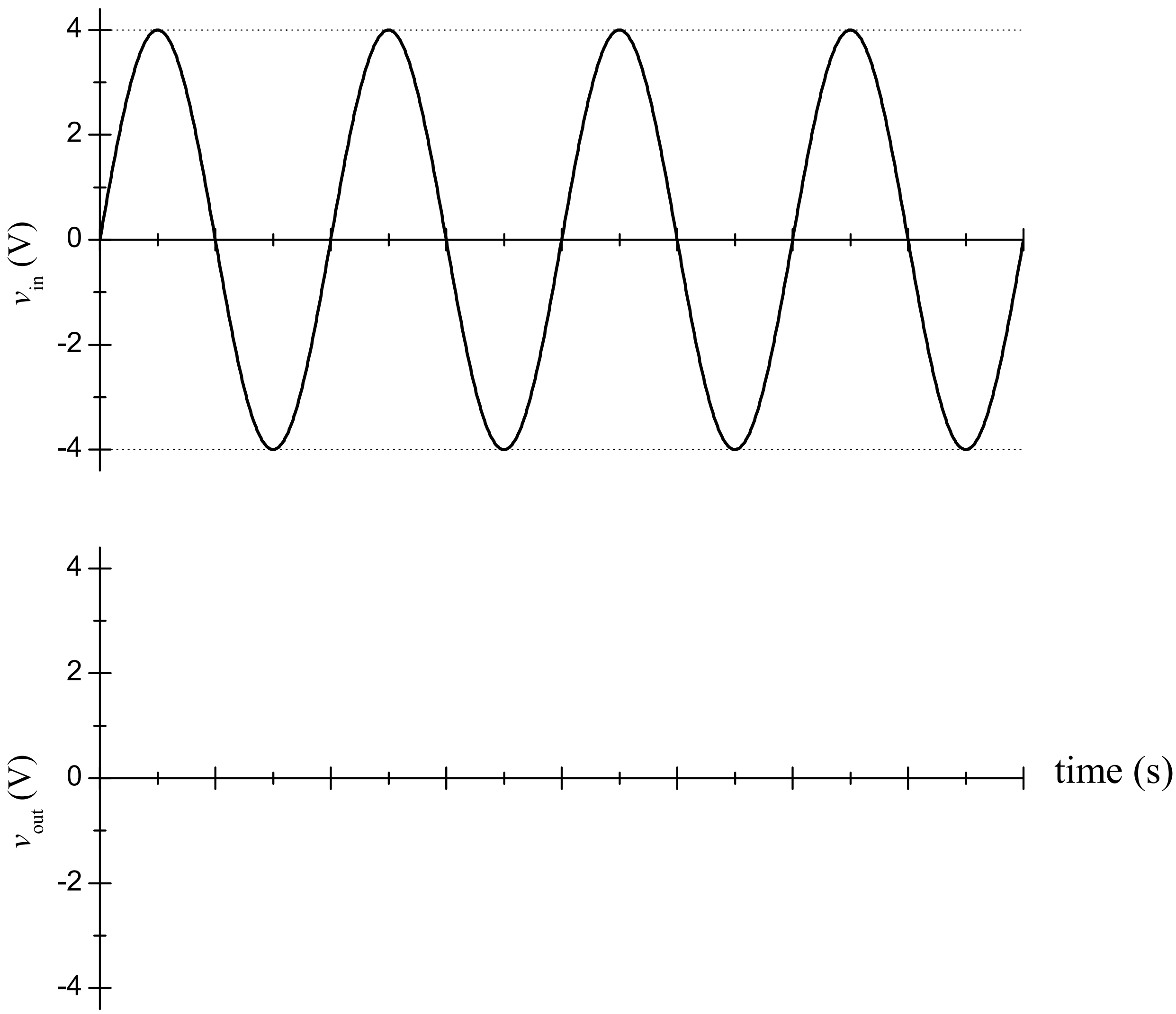
(10pts) **6.** The circuit below consists of a sinusoidal input v_{in}, four diodes, and a resistor R_{L}. The output of the circuit v_{out} is taken to be the voltage across R_{L}.



(a) The next page shows a plot of v_{in} versus time. On the axes directly below, sketch a plot of v_{out} versus time. Draw your plot to scale as best you can. (7 marks)

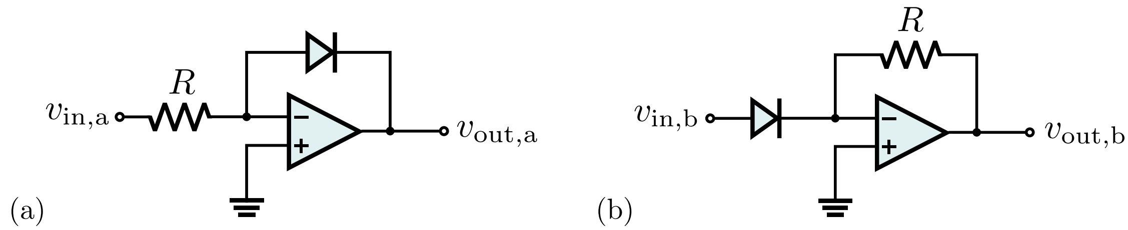
(b) With the addition of a single capacitor C, this circuit can be made into an AC-to-DC converter. Add the required capacitor to the circuit above. How should the product $R_{\text{L}}C$ compare to the period T of the AC input v_{in} in order for the circuit to work as an effective AC-to-DC converter? (3 marks)

10 pts



0 pts

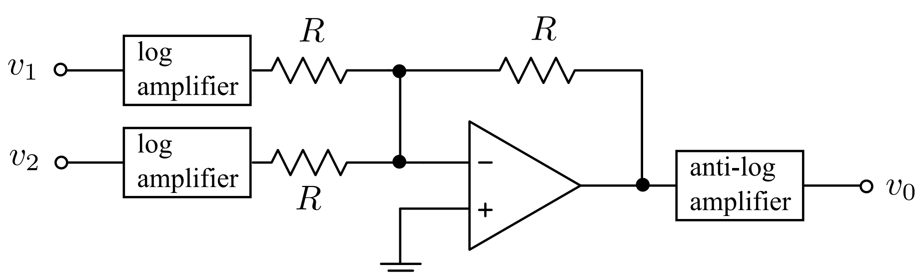
(10pts) **7.** Figures (a) and (b) below show the log- and the anti-log-amplifiers respectfully.



For suitable $v_{\text{in,a}}$ and $v_{\text{in,b}}$, the outputs of the circuits are given by:

$$v_{\text{out,a}} = -\frac{k_B T}{e} \ln \frac{v_{\text{in,a}}}{I_0 R}$$
$$v_{\text{out,b}} = -I_0 R \exp\left(\frac{e\, v_{\text{in,b}}}{k_B T}\right)$$

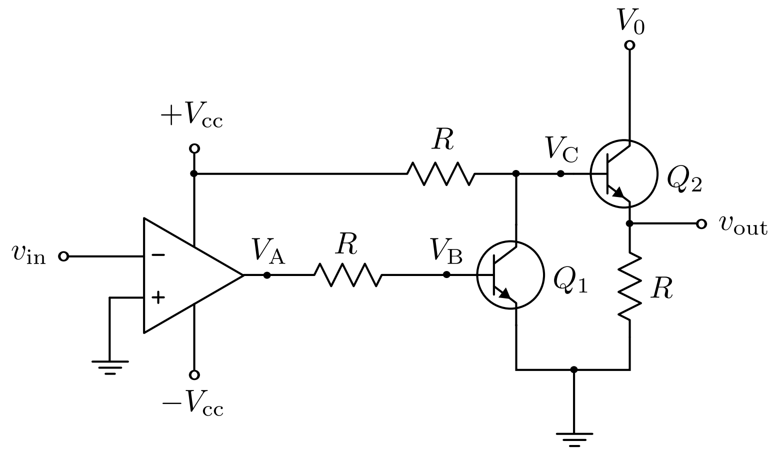
The circuit below makes use of the log and anti-log amplifiers. In this circuit the box labelled "log amplifier" corresponds to circuit (a) above and the box labelled "anti-log amplifier" corresponds to circuit (b) above. Assume that all resistors used in this problem are identical and that all diodes used in this problem are identical.



Find an expression for v_0 of this circuit. What kind of function does this circuit perform?

10 pts

(10[pts]) **8.** The circuit shown below is similar (but not identical) to a circuit that you built and studied in Experiment #6.



Assume that $R = 10$ kΩ, $V_{cc} = 15$ V, and $V_0 = 5$ V. In this problem you are simply asked to complete the table shown below. For the input voltages v_{in} shown in the table, enter the resulting values for V_A, V_B, V_C, and v_{out}. In the Q_1 and Q_2 columns indicate whether each transistor is conducting or not conducting. If a transistor is in its conducting state, enter "ON". If a transistor is not conducting, enter "OFF".

v_{in} (V)	V_A (V)	V_B (V)	V_C (V)	Q_1	Q_2	v_{out} (V)
-3						
-1						
$+1$						
$+5$						

10 pts